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MAGNETIC AMPLIFIER SERVO COMPENSATION

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U. S. NAVAL ORDNANCE LABORATORY
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MAGNETIC AMPLIFIER SERVO COMPENSATION

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ABSTRACT: Techniques are described whereby a half-wave, bridge-type magnetic amplifier with integral feedback forms a versatile servo building block. Proper adjustment of the polarity and amplitude of the integral feedback makes this circuit a lead network, a lag network, or an integrator. In addition to these characteristics, the circuit can be used to modulate, demodulate, or amplify a-c or d-c, merely by selecting the proper output component. This circuit is useful with vacuum tube amplifier servo controllers as well as with magnetic amplifier servo controllers.

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The use of magnetic amplifiers in servo systems with performance characteristics far exceeding those of previous magnetic amplifier servo systems required new methods of stabilization and compensation.

Under the Magnetic Amplifier Servo Systems Development Program, NOL-Re4a 78-2-53, new compensation techniques were developed. The derivations and experimental checks are contained in this report.

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Commander

D. S. MUZZEY, Jr.
By direction

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ILLUSTRATIONS

MAGNETIC AMPLIFIER SERVO COMPENSATION

INTRODUCTION

1. The half-wave, bridge-type magnetic amplifier[1] responds equally well to a-c (modulated carrier) or d-c (modulation-frequency) signals; and the output is phase-reversible, pulsating d-c. Both the d-c and a-c components of the output are linearly related to control voltage. The gain of interest in the following derivations is the d-c gain (d-c volts out divided by d-c volts in). This steady state d-c gain will be designated K_D.

2. This half-wave amplifier has a time delay dependent only on the number of stages[1]; therefore, the amplifier transfer function for voltages of modulation frequency, ω, is:

$$K_A = K_D e^{-T_A j\omega} \qquad \text{Eq. (1)}$$

where T_A is the time constant determined by the number of stages in the amplifier. This time constant is one cycle of the supply frequency for the first stage and one-half cycle for each additional stage. For a two-stage, 400-cycle amplifier the time constant is:

$$T_A = 3.75 \text{ millisecond} \qquad \text{Eq. (2)}$$

3. In the region of frequencies in which the phase angle, $T_A\omega$, is small the assumption can be made that

$$K_A = K_D \qquad \text{Eq. (3)}$$

In a two-stage, 400-cycle amplifier, the phase shift at a signal frequency of 100 radians per second is approximately 20 degrees. For frequencies below 100 radians per second, the phase shift is assumed to be ero, remembering that some small error will be introduced by this assumption.

4. The gain of a half-wave, bridge type amplifier is greatest when the control source impedance is the smallest possible. For many servo applications, the amplifier must be operated from a control transformer. The load impedance on the output of the control transformer is frequently required to be at least 10,000 ohms. Amplifiers of the above type are usually designed, therefore, with a 10,000 ohm resistor in series with the input windings in which case the d-c input resistance is very nearly 10,000 ohms since the d-c resistance of the control windings is usually much smaller than this value.

5. When the output of the amplifier is fed back through a resistance capacitance network as shown in figure 1, only the d-c component of the amplifier output will appear across the capacitor if the RC time constant is long compared to the period of the supply voltage and the amplifier input resistance does not load the RC network too severely. When there is some loading of the network by the amplifier input resistance the feedback function will be changed from that of a simple RC network. Whether or not there is loading of the network the capacitor voltage is the input voltage to the amplifier from the feedback network; therefore, from figure 2 it is seen that the feedback function of figure 1 is:

$$\frac{E_f}{E_o} = \frac{\alpha}{\alpha T j\omega + 1} \qquad \text{Eq. (4)}$$

where $\alpha = R_c/(R+R_c)$, $T = RC$, and R_c is the amplifier input resistance. Using the above value of feedback function, the closed loop transfer function of figure 1 is:

$$\frac{E_o}{E_i} = \frac{K_D(\alpha T j\omega + 1)}{\alpha T j\omega + 1 \mp \alpha K_D} \qquad \text{Eq. (5)}$$

Lead Network

6. When the feedback of figure 1 is negative, equation (5) reduces to:

$$\frac{E_o}{E_i} = \frac{\frac{K_D}{1+\alpha K_D}\,(\alpha T j\omega + 1)}{\frac{\alpha T}{1+\alpha K_D}\, j\omega + 1} \qquad \text{Eq. (6)}$$

This is a lead circuit[2] with lower break frequency $1/\alpha T$, break-frequency spread $(1+\alpha K_D)$, and zero frequency gain $K_D/(1+\alpha K_D)$. This value of zero-frequency gain is based on the assumption that the only usable portion of the output is the d-c component. The output of this network is actually pulsating d-c; consequently, when the output of this network is fed directly into the input of a half-wave amplifier, which responds to both d-c and fundamental a-c components, the zero frequency gain will be somewhat greater than that indicated by equation (6).

7. Since this network will respond equally well to a-c or d-c inputs, and since the output contains both a-c and d-c components, it is a useful and versatile servo building block. By using the appropriate component of the output for a given input, a-c or d-c, it is seen that this network can be used as a modulator, demodulator, a-c amplifier, or d-c amplifier each with the same lead characteristic given by the frequency-variant portion of equation (6).

8 Shown in figure 3 are calculated and measured curves of the frequency variant portion of equation (6). The agreement is very good except at high frequencies where the phase shift in the amplifier, neglected in the analysis, introduces considerable error. From the results of figure 3, it is seen that the characteristics of such a network can be predicted using simple servo theory with sufficient accuracy for most servo design problems. Such a network has been successfully used to compensate a practical servo system[3].

9. One caution must be observed when using this lead network. When the phase shift around the loop is 180 degrees, the loop gain must be less than one to insure stability. The complex loop gain is:

$$\frac{E_f}{E_e} = \frac{\alpha K_D e^{-T_A j\omega}}{\alpha T j\omega + 1} \qquad \text{Eq. (7)}$$

For a practical circuit, 180 degrees phase shift will occur where:

$$\alpha T\omega \gg 1 \qquad \text{Eq. (8)}$$

in which case the phase shift of the denominator of equation (7) is very nearly $\pi/2$ radians. In view of this, the phase shift of 180 degrees will occur when:

$$T_A\omega = \frac{\pi}{2} \qquad \text{Eq. (9)}$$

At the frequency determined by equation (9) the magnitude of E_f/E_e must be less than one, hence:

$$\frac{K_D}{T\omega} < 1 \qquad \text{Eq. (10)}$$

Equations (9) and (10) set the lower limit on the time constant, T, that can be used with a given amplifier gain, K_D, to insure a stable lead network.

Lag Network

10. When the feedback in figure 2 is made positive and the zero-frequency loop gain, αK_D, is less than one, equation (5) becomes:

$$\frac{E_o}{E_i} = \frac{\frac{K_D}{1-\alpha K}(\alpha T j\omega + 1)}{\frac{\alpha T}{1 \alpha K_D} j\omega + 1} \qquad \text{Eq. (11)}$$

This is seen to be a lag network with upper break frequency $1/\alpha T$, break-frequency spread $1/(1 \alpha K_D)$, and zero frequency gain $K_D/(1 - \alpha K_D)$.

11. This lag network, like the lead network discussed previously, is a versatile servo building block. For the reasons given previously, this lag network can be used for modulation, demodulation, a-c amplification, or d-c amplification.

12. As before, the zero-frequency gain given by equation (11) is on the basis of the d-c component of the output. If both d-c and a-c components of the output are used, as in driving another magnetic amplifier, the zero-frequency gain will be somewhat higher than that indicated by equation (11).

13. If a given amplifier has sufficient gain that αK_D is greater than one, the circuit configuration of figure 4 can be used to obtain the proper lag characteristic while keeping the maximum possible zero-frequency gain. In this case, referring to figure 4, the feedback function is now:

$$\frac{E_f}{E_e} = \frac{\alpha \alpha_1}{\alpha T j\omega + 1} \qquad \text{Eq. (12)}$$

where $\alpha_1 = R_2/(R_1+R_2)$. Of course, the total resistance (R_1+R_2) must be much less than R for the above expression to hold. The output impedance of the magnetic amplifier is sufficiently low that this condition is easily met in any practical case. Using the circuit of figure 4, we see that the transfer function is now:

$$\frac{E_o}{E_i} = \frac{\frac{K_D}{1-\alpha\alpha_1 K_D}\,(\alpha T j\omega + 1)}{\frac{\alpha T}{1-\alpha\alpha_1 K_D}\, j\omega + 1} \qquad \text{Eq. (13)}$$

The details of this lag function are very similar to those of equation (11).

14. Shown in figure 5 are measured and calculated curves of a typical lag function as given by equation (11) or (13). Once again the agreement between curves is good except that for higher frequencies the error between calculated and measured phase angles becomes appreciable due to the assumption of negligible amplifier phase shift in the calculations. This circuit has been used successfully to raise the velocity constant of a servo without appreciably affecting the servo bandwidth.

Integrator

15. When the feedback of figure 4 is positive and the factor $\alpha\alpha_1 K_D$ is equal to one, the over-all transfer function from equation (5) is:

$$\frac{E_o}{E_i} = \frac{K_D(\alpha T j\omega + 1)}{\alpha T j\omega} \qquad \text{Eq. (14)}$$

For frequencies, ω, such that

$$\alpha T\omega << 1, \qquad \text{Eq. (15)}$$

the circuit described by equation (14) is an integrator with the transfer function:

$$\frac{E_o}{E_i} = \frac{K_D}{\alpha T j\omega} \qquad \text{Eq. (16)}$$

When a steady signal is put into this circuit, the output, being proportional to the integral of the input signal, will build up until the amplifier saturates. Hence, a constant output voltage is obtained from this circuit only when the input voltage is zero. This type of transfer function when placed in a servo loop with the proper types of stabilization will yield a zero-velocity-error system. Such a system, having the block diagram shown in figure 5, has been successfully synthesized.

Conclusion

16. The techniques described above are useful in vacuum tube servo systems as well as in magnetic amplifier servo systems. The usefulness is enhanced by the ease with which circuit characteristics can be computed with sufficient accuracy for design purposes.

17. When these design methods are used with half-wave bridge-type magnetic amplifiers the resulting compensation networks can be used with any combination of a-c and/or d-c input, and a-c and/or d-c output. These networks are only as sensitive to line voltage and frequency variations as the amplifier used in the network. When the amplifier components are properly matched, the amplifier is extremely insensitive to these variations.

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2. H. R. Ahrendt and J. F. Taplin, "Automatic Feedback Control," McGraw-Hill Book Company, Inc., New York, N. Y., 1951.

3. H. H. Woodson, A. E. Schmid, C. V. Thrower, "Compensation of a Magnetic Amplifier Servo System," paper presented at the National Electronics Conference, Chicago, Illinois, 1952.

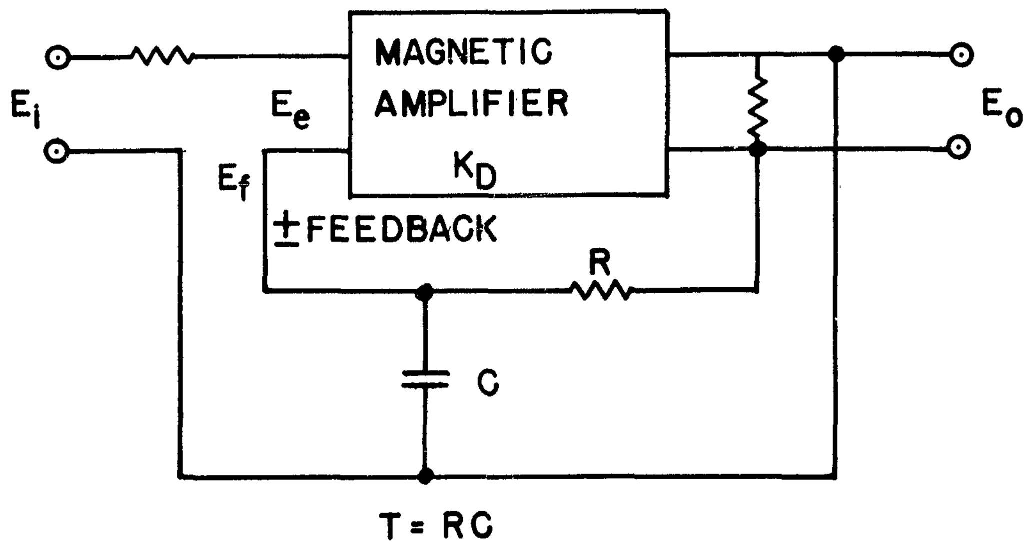


FIGURE 1

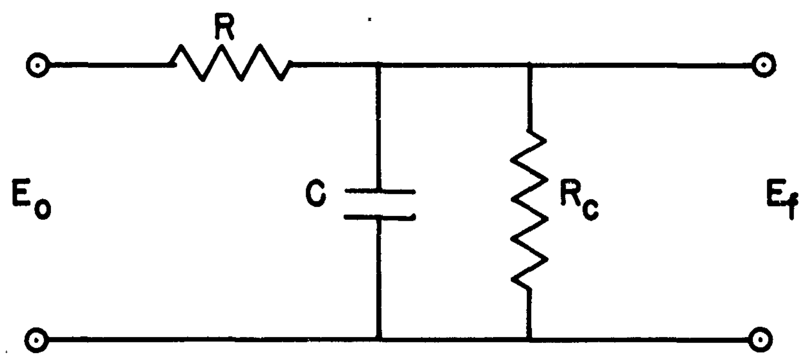


FIGURE 2

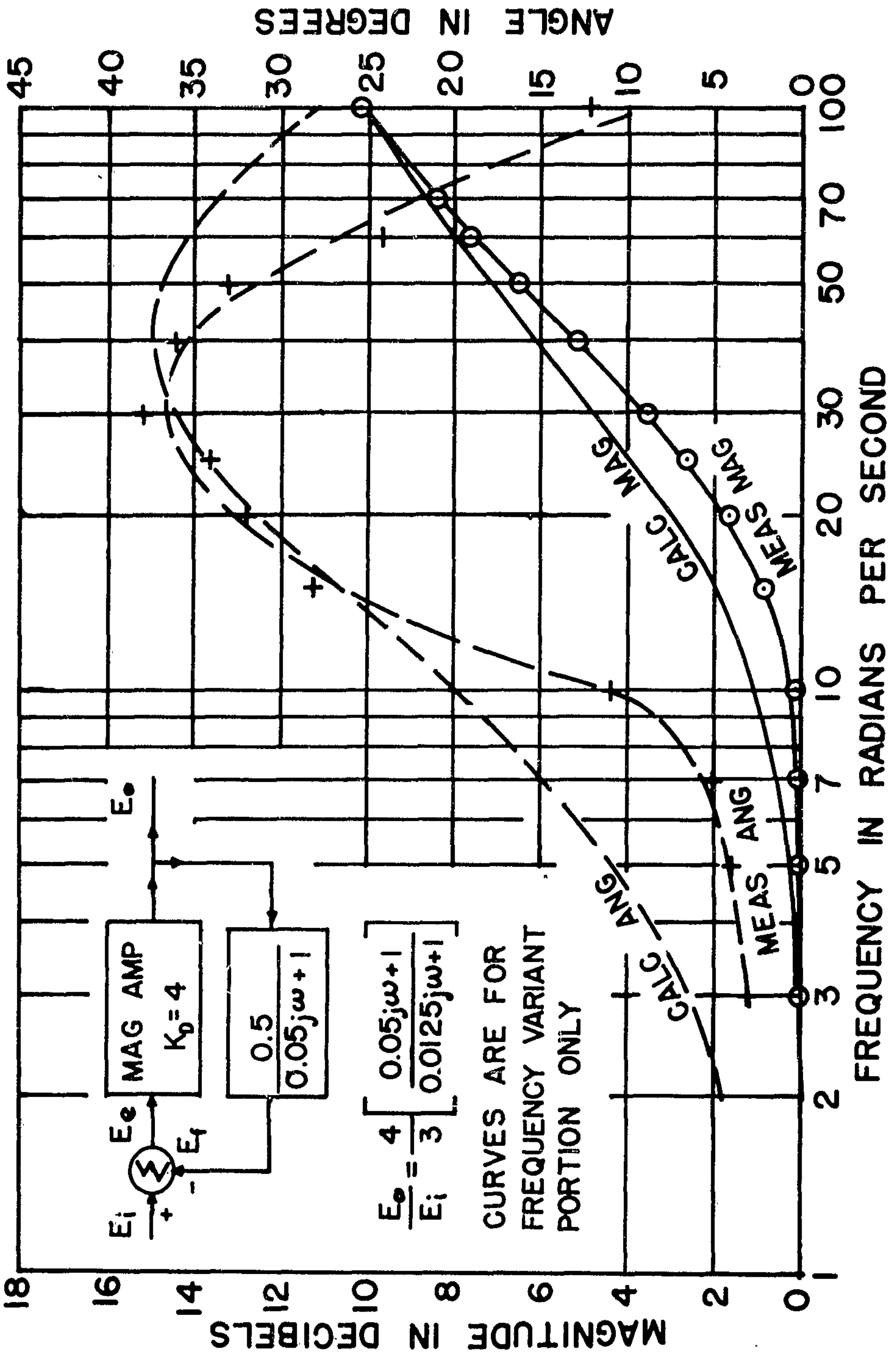
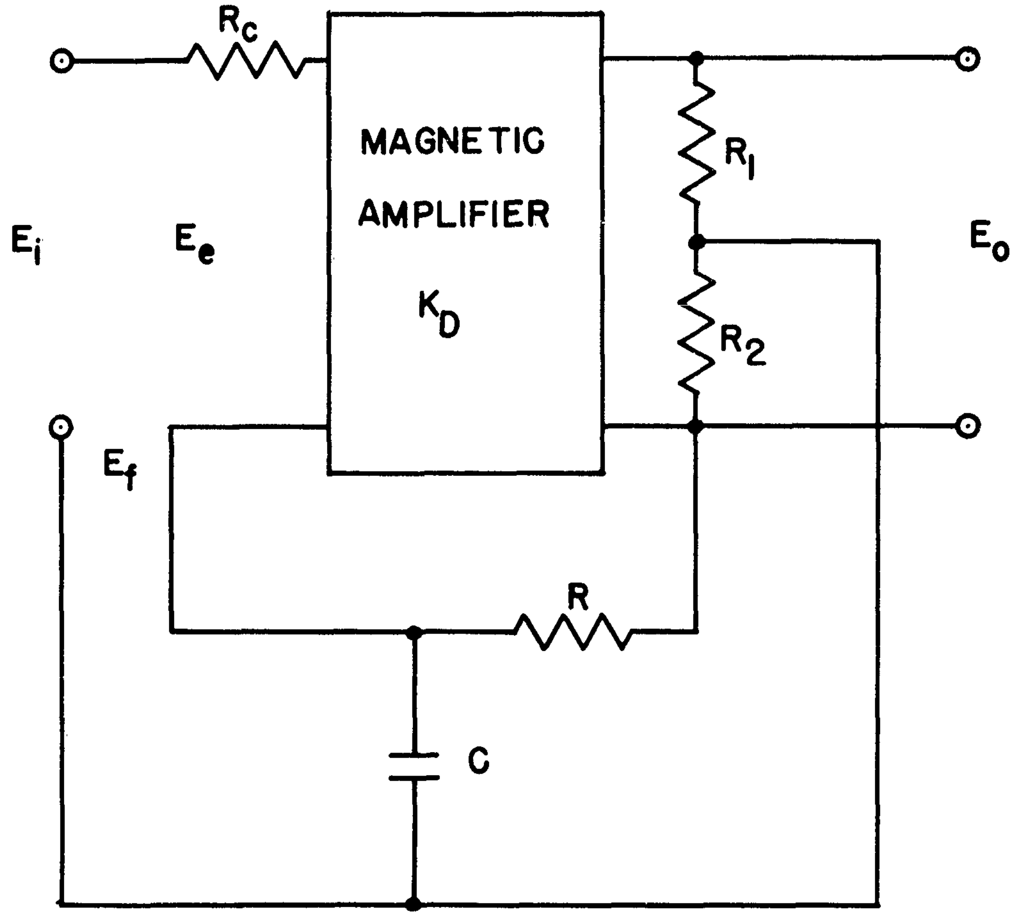
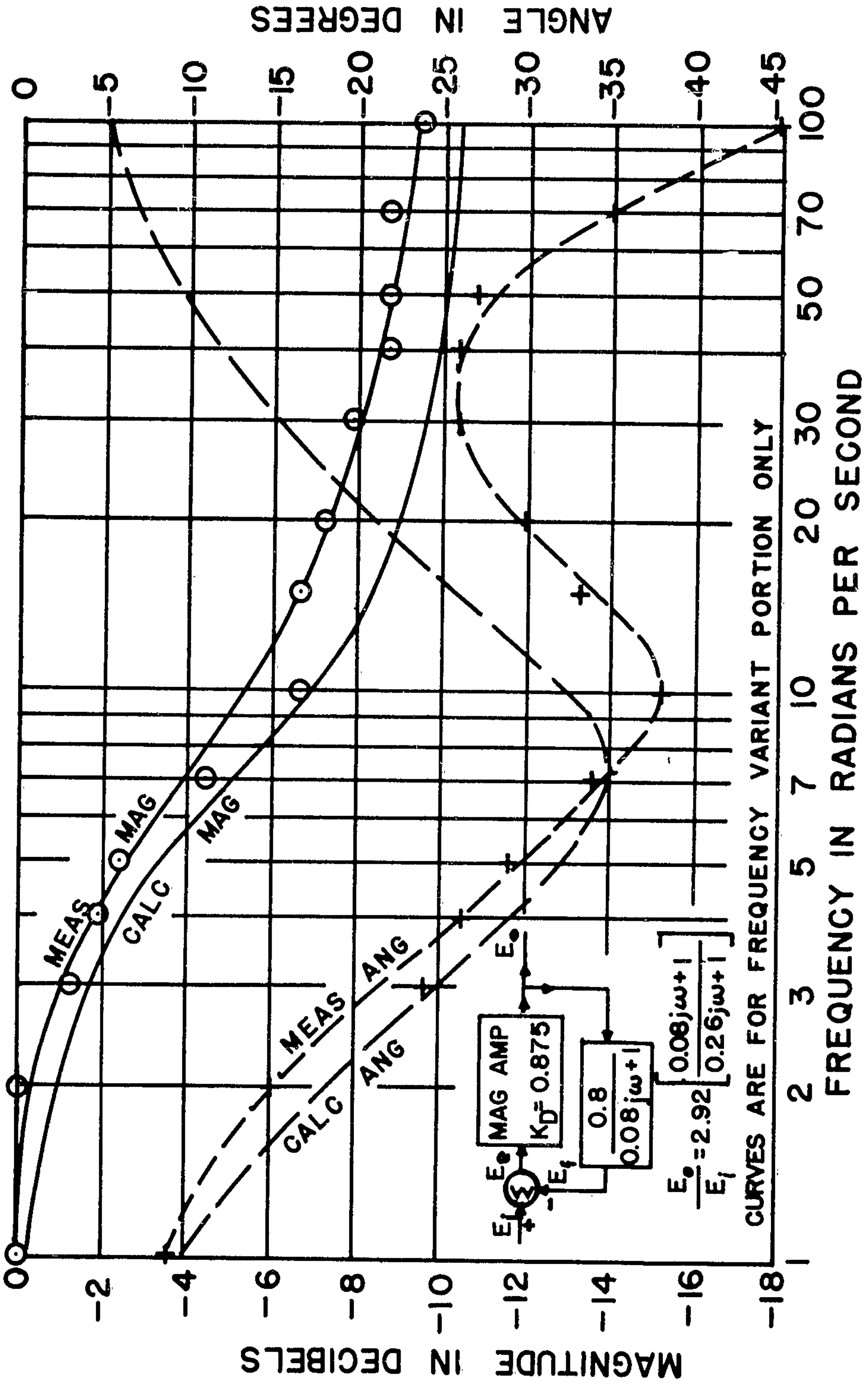

ANGLE IN DEGREES
MAGNITUDE IN DECIBELS
FREQUENCY IN RADIANS PER SECOND
CALC MAG
MEAS MAG
CALC ANG
MEAS ANG
MAG AMP
$K_D = 4$
$\frac{0.5}{0.05j\omega+1}$
E_i
E_e
E_f
E_o
$\frac{E_o}{E_i} = \frac{4}{3}\left[\frac{0.05j\omega+1}{0.0125j\omega+1}\right]$
CURVES ARE FOR
FREQUENCY VARIANT
PORTION ONLY

FIGURE 3



$T = RC$ $\qquad \alpha_1 = \dfrac{R_2}{R_1 + R_2}$ $\qquad \alpha = \dfrac{R_c}{R_c + R}$

FIGURE 4


ANGLE IN DEGREES
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CURVES ARE FOR FREQUENCY VARIANT PORTION ONLY
MEAS MAG
CALC MAG
MEAS ANG
CALC ANG
MAG AMP
$K_D = 0.875$
$\frac{0.8}{0.08j\omega+1}$
$\frac{E_o}{E_i} = 2.92 \left[\frac{0.08j\omega+1}{0.26j\omega+1}\right]$

FIGURE 5

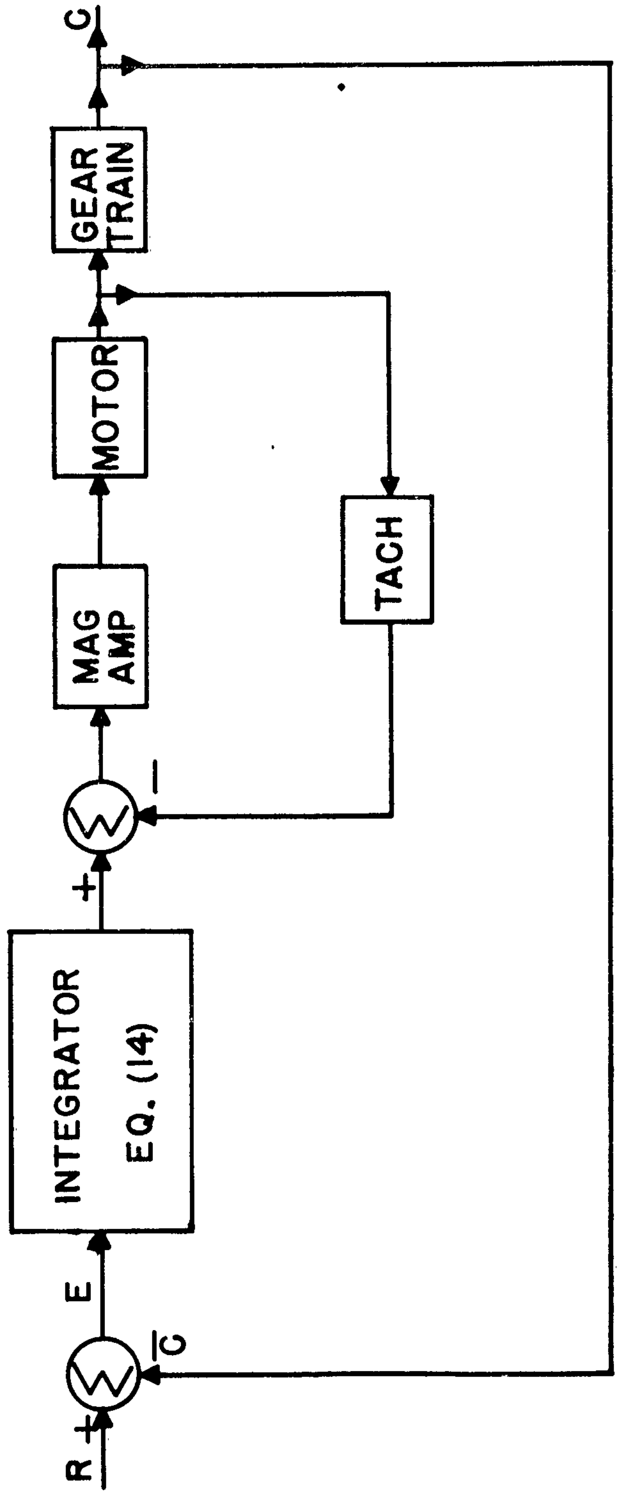
R
+
E
$\overline{C}$
INTEGRATOR
EQ. (14)
+
−
MAG
AMP
MOTOR
TACH
GEAR
TRAIN
C

FIGURE 6

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